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Chapter 12 Using Analogies as a Basis for Teaching Cognitive Readiness

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An adaptive problem solver needs to be capable of rapidly analyzing novel situations and planning a course of action that is likely to be effective in achieving goals (see Mayer, this book, for additional information on adaptive problem solving). Cognitive readiness, broadly construed, is the state of being prepared to apply one's prior knowledge in an adaptive manner. In this chapter we will review what is known about the role of *analogical reasoning* in fostering cognitive readiness. Analogical reasoning is the process of identifying goal-relevant similarities between a familiar *source* analog and a novel, less understood *target*, and then using the correspondences between the two analogs to generate plausible inferences about the latter (Holyoak, 2012).

The source may be a single concrete situation (e.g., the naval blockade of Cuba in 1962, imposed by the United States in response to placement of nuclear missiles there), a set of multiple cases (e.g., other specific examples of naval blockades), or a more abstract schema (e.g., naval blockades in general). The target may be a situation from a relatively similar domain (e.g., uranium enrichment in Iran, which might be countered by a naval blockade) or a situation in a remote domain (e.g., a computer site launching a cyber attack, which might be countered by an electronic "blockade" designed to cut the threatening site off from the Internet). Cognitive readiness implies that the learner is prepared to transfer knowledge from the

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source(s) to a target when the context has been altered, a substantial delay has ensued, and/or the solution applied in the source requires substantial modification to be applicable in the target.

12.1 The Role of Causal Models in Analogical Transfer

Cognitive readiness, and hence successful transfer of knowledge, ultimately depends on the learner's understanding of the source domain. Based on analyses of the use of analogy to support hypotheses in areas of science and the law, Bartha (2010) argues that the reasoner's understanding of the source places an upper bound on the support that the source can provide for a hypothesis about the target. For empirical knowledge (roughly, knowledge about how the world works), understanding can be viewed as the acquisition of a *causal model* of the source domain (Holyoak, Lee, & Lu, 2010; Lee & Holyoak, 2008; Waldmann & Holyoak, 1992). Intuitively, transfer depends on evaluating whether the source and target are alike in terms of the factors that enter into cause-effect relations that impact on goal attainment. For example, the naval blockade of Cuba proved effective in part because Cuba is an island, making it especially dependent on sea transport. If this source analog were used to support a proposal for a naval blockade of Iran, the analogical argument would be weakened by the fact that Iraq is attached to the mainland (as well as by other causally relevant factors, such as Iraq's relatively large size). In contrast, the analogical argument would not be seriously weakened by various other differences between Cuba and Iraq (e.g., the former has a communist government, the latter is an Islamic theocracy) that lack any causal connection to the effectiveness of a naval blockade.

It has long been recognized that understanding the causal structure of the source is critical in producing analogical inferences that enable solving a problem in a novel target. Holyoak (1985) emphasized the centrality of pragmatic constraints on analogical inference that operate in service of goal attainment during problem solving: "the goal is a *reason* for the solution plan; the resources *enable* it; the constraints *prevent* alternative plans; and the outcome is the *result* of executing the solution plan" (p. 70). A well-understood source analog can provide detailed information about a specific pattern of causal events critical to goal attainment, which can be used to provide guidance in making inferences about a complex and poorly understood target analog (see Holyoak & Thagard, 1995).

A series of experiments reported by Lee and Holyoak (2008) demonstrated how causal knowledge guides analogical inference, and that analogical inference is not solely determined by quality of the overall mapping between source and target. Using a common-effect structure (multiple possible causes of a single effect; see Waldmann & Holyoak, 1992), Lee and Holyoak manipulated structural correspondences between the source and the target as well as the causal polarity (generative or preventive) of multiple causes present in the target. In Fig. 12.1, the three panels show examples of causal structures used in their experiments. In the source, three causes (two generative, G_1 and G_2, and one preventive, P) are simultaneously

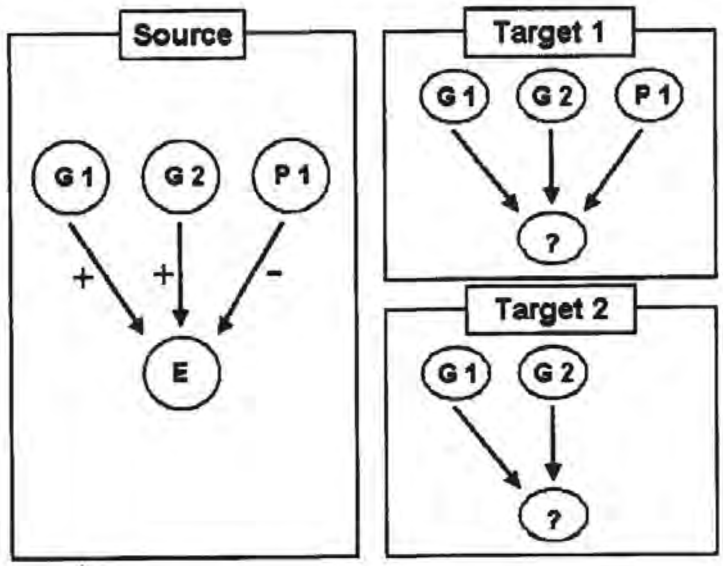

Fig. 12.1 Example of use of causal models in analogical inference. G, P, and E represent a generative cause, preventive cause, and effect, respectively (from Lee & Holyoak, 2008; reprinted by permission)

present, and when the influences of these three causes are combined, the effect occurs. Target analog 1 (G_1G_2P condition) shares all three causal factors with the source, whereas target 2 (G_1G_2 condition) shares only the two generative factors with the source, not the preventive one. Accordingly, target 1 has greater overlap with the source than does target 2, where "overlap" can be defined in terms of some measure of semantic similarity of elements and/or structural matches based on corresponding relations (e.g., Gentner, 1983). All previous computational models of analogical inference (e.g., Falkenhainer, Forbus, & Gentner, 1989; Holyoak & Thagard, 1989; Hummel & Holyoak, 1997, 2003), which assume that the plausibility of target inferences increases monotonically with the overall overlap between the source and target analogs, therefore predict that target 1 is more likely than target 2 to have the effect E.

If analogical inference is guided by causal models, however, the prediction reverses, because dropping a preventive cause, as in target 2 relative to target 1, yields a causal model of the target in which the probability that the effect occurs will *increase*. Lee and Holyoak (2008) tested these alternative predictions using both within-domain analogs (examples of a type of imaginary animal) and cross-domain analogs (imaginary systems in chemistry and astronomy). For both types of materials, they found that people in fact rated analogs in the form of target 2 (G_1G_2) as more likely to exhibit the effect than analogs in the form of target 1 (G_1G_2P; see Fig. 12.2, bottom panel), even though participants rated the target in the G_1G_2 condition as less similar than that in the G_1G_2P condition to the source analog (Fig. 12.2, top panel). Because analogical inferences are based on shared causal structure, such inferences are partially dissociable from overall similarity of the analogs. These findings suggest that understanding human use of analogy to make inferences requires deeper consideration of how causal knowledge is integrated with structural mapping. The results obtained by Lee and Holyoak (2008) imply that people have a natural propensity to focus on causal structure when they use analogies as a source of inferences about an unfamiliar target situation.

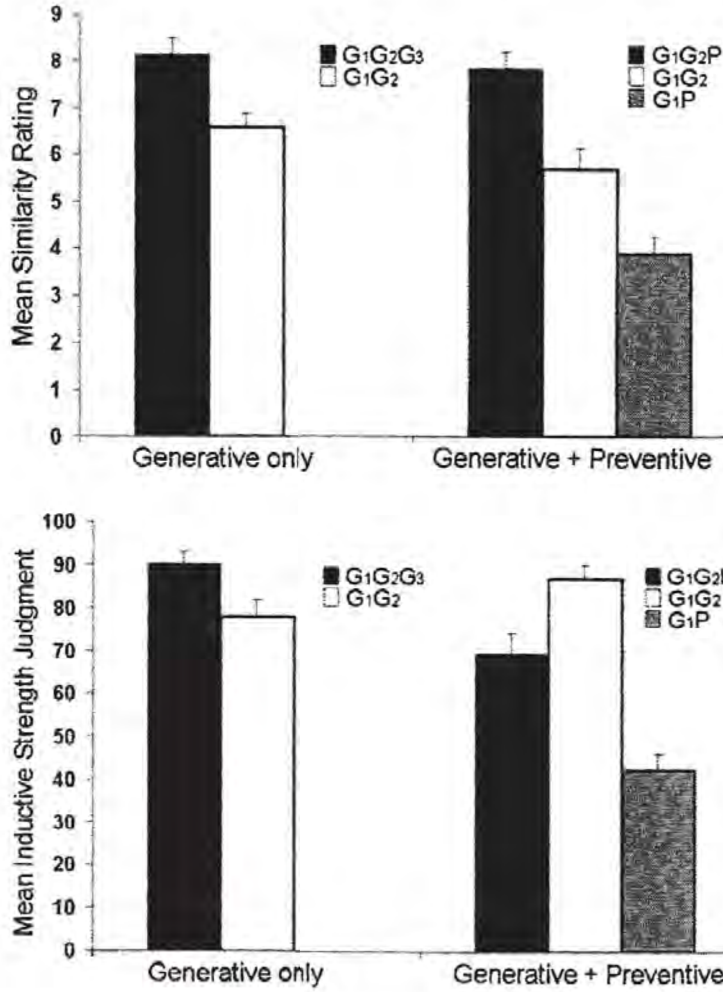

Fig. 12.2 Mean similarity ratings (*top*) and mean inductive strength judgments (*bottom*) for each argument type in the generative-only and generative + preventive conditions of Lee and Holyoak (2008, Experiment 1). *Error bars* represent 1 SEM (from Lee & Holyoak, 2008; reprinted by permission)

Holyoak et al. (2010) developed a computational model of how analogical inference is guided by causal models, formalized within the framework of Bayesian inference. The conceptual framework is schematized in Fig. 12.3. The key assumption of the model is that analogical reasoning uses causal knowledge of the source to develop a causal model of the target, which can in turn be used to derive a variety of inferences about the values of variables in the target. The first stage of analogical inference is learning a causal model of the source (step 1 in Fig. 12.3). The source model is then mapped to the initial (typically impoverished) representation of the target. Based on the mapping (step 2), the causal structure and strengths associated with the source are transferred to the target (Step 3), creating or extending the causal model of the latter. The model of the target can then be "run" (step 4), using causal reasoning to derive inferences about the values of endogenous variables in the target.

For simple causal networks based on binary variables (i.e., cause and effect factors can be either present or absent), Holyoak et al.'s (2010) theory of analogical inference adopts the independently supported assumptions of the Bayesian extension of the power PC theory ("power theory of the probabilistic contrast model": Cheng, 1997; Lu, Yuille, Liljeholm, Cheng, & Holyoak, 2008). The power PC theory assumes that people use a normative process to integrate the influences of multiple potential causes that co-occur. Holyoak et al. show that their Bayesian model

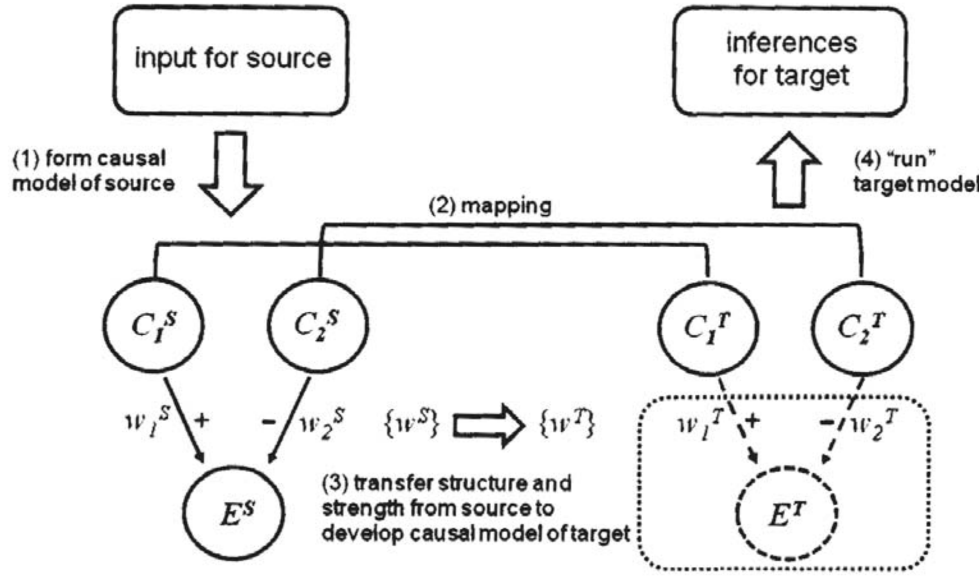

Fig. 12.3 Framework for analogical transfer based on causal models. *Dotted lines* indicate knowledge transferred from source to target (see text) (from Holyoak et al., 2010; reprinted by permission)

provides a close qualitative fit to human judgments for a variety of causal inferences about a target analog based on transfer from a source analog.

The causal-model approach to analogical inference highlights some basic constraints that have implications for cognitive readiness. In particular, the quality of analogical inferences is ultimately limited by the reasoner's causal understanding of the source (Bartha, 2010). If the reasoner fails to understand (or misunderstands) the causal structure of the source, analogical inferences will inevitably suffer. At the same time, transfer is also limited by the quality of the mapping between the source and target. When the correspondences between the analogs are unclear, transfer will also be limited. Critically, not all correspondences are equally important. Rather, correspondences between elements causally related to a reasoning goal are central (Holyoak, 1985; Spellman & Holyoak, 1996). We will now consider some implications of these constraints on analogical transfer for training techniques that can foster readiness to use analogies effectively.

12.2 Analogical Comparison as a Learning Mechanism

Given that analogical transfer is ultimately limited by the learner's understanding of the source domain, it follows that readiness for transfer depends on training techniques that enhance causal understanding of the source. It turns out that one of the most effective techniques for fostering understanding of the source domain is to provide multiple examples and encourage analogical reasoning about the relationships between them. By a kind of "analogical bootstrapping," analogical reasoning can lead to the induction of a more abstract schema, which in turn enables more adaptive transfer.

In an early study of the role of analogical comparison in learning, Gick and Holyoak (1983) employed analogous "convergence" problems as training and transfer tasks. Convergence problems are based on Duncker's (1945) "radiation problem," in which the reasoner must find a way to destroy a stomach tumor without destroying surrounding healthy tissue, by using a type of ray that at sufficiently high intensity will destroy the tumor, but at that same intensity would also destroy the surrounding tissue through which the ray must pass to reach the tumor. The convergence solution requires directing multiple, converging rays toward the tumor at different angles, with the intensity of each ray being sufficiently low to avoid destruction of the surrounding tissue, but the combined intensity of the rays being sufficiently high to destroy the tumor (Gick & Holyoak, 1980).

Gick and Holyoak (1983) trained college students on either one or two convergence problem analogs and tested for transfer with a second analog. An example of an analog for the radiation problem involves a general who wants to amass his forces to attack a fortress, but all the roads leading to the fortress contain mines that will detonate if a sufficiently large group traverses the road. A second analog might involve a fire-fighting scenario in which multiple small hoses are used to extinguish a centrally located fire. In the two-analog condition, participants were asked to compare the two source analogs and write down their commonalities. Gick and Holyoak found that those participants who compared two source analogs exhibited substantially more transfer than did those who saw just one source analog. Those participants who explicitly stated the key aspects of the convergence solution (small forces applied simultaneously from multiple directions) in describing the similarities between the analogs were especially likely to show spontaneous transfer to the tumor problem. Transfer was further enhanced when the concrete analogs were accompanied by a representation of the general solution principle (either as a verbal statement or as a simple diagram). Interestingly, neither the verbal statement nor the diagram conveyed any additional benefit when paired with a single analog.

Gick and Holyoak (1983) argued that encouraging comparison of multiple source analogs (particularly when accompanied by a representation of the solution principle) fostered abstraction of a generalized schema for the problem category, improving the likelihood that subjects will spontaneously recognize the structural similarities among the problems and thereby facilitating transfer (see also Ross & Kennedy, 1990). The critical contribution of analogical comparison to learning is that it highlights the commonalities between two analogs, focusing attention on key aspects causally related to goal attainment. For complex problem situations, the search space of features and relations that may be relevant to a solution will be extremely large. By making an analogical comparison between two examples that mainly share key structural relations, while differing in more surface aspects, the learner will be led to focus on structural relations that might otherwise have been overlooked. The benefit of learning by comparison can be further increased by providing more than two source analogs (Catrambone & Holyoak, 1989).

Very similar results have been obtained in studies in which business students were trained in negotiation strategies (Gentner, Loewenstein, & Thompson, 2003; Loewenstein, Thompson, & Gentner, 1999; Thompson, Gentner, & Loewenstein

2000). These investigators found that instructions to compare two cases and write down their commonalities (instructions very similar to those used by Gick & Holyoak, 1983) resulted in substantial transfer when the students later engaged in an actual face-to-face bargaining session. Moreover, the benefit of comparison was about three times greater than that provided by presenting two cases (even on the same page) without comparison instructions (Gentner et al., 2003; Thompson et al., 2000; see also Kurtz, Miao, & Gentner, 2001). As in the Gick and Holyoak (1983) study, the quality of the principles elicited during the comparison predicted subsequent transfer.

For much simpler problems, the benefit of comparison as a learning strategy has been demonstrated in children as young as 3 years old (Loewenstein & Gentner, 2001). An important refinement of the use of comparison as a training technique is to provide a series of comparisons ordered "easy to hard," where the early pairs share salient surface similarities as well as less salient relational matches and the later pairs share only relational matches. This "progressive alignment" strategy serves to promote a kind of analogical bootstrapping, using salient similarities to aid the learner in identifying appropriate mappings between objects that also correspond with respect to their relational roles (Kotovsky & Gentner, 1996).

In general, the fundamental benefit of analogical comparison is to foster the generation of a more abstract representation of a class of situations that share important causal structure. When teaching types of problems, a closely related training strategy is to focus students' attention on the explicit subgoal structure of the problems, so that they represent the reasons why actions were performed (Catrambone, 1995, 1996, 1998; Catrambone & Holyoak, 1990). Acquiring explicit knowledge of subgoal structure aids cognitive readiness by making it easier to adapt old solutions from related but non-isomorphic transfer problems (e.g., by deleting, re-ordering, or modifying individual steps in a solution). Teaching subgoal structure has been shown to be especially important in fostering transfer to problems that are relatively dissimilar to the cases used in training (Catrambone, 1998).

12.3 Limits in Processing Resources

The work on learning from comparisons and from cues to subgoal structure demonstrates that simply presenting cases does not suffice to guarantee that learners will be cognitively ready for subsequent transfer to novel problems that share the underlying structure of the training examples. Both developmental studies (Richland, Morrison, & Holyoak, 2006) and neuroimaging studies with young adults (Cho et al., 2010) reveal that analogical mapping is dependent on key executive functions, especially working memory and inhibitory control. Learning from and by analogy thus requires that instructors ensure analogical learning opportunities do not overtax background knowledge, adequate working memory resources, or ability to avoid distraction from surface similarity.

Working memory and inhibitory control are critically involved in two aspects of analogical reasoning: representing and integrating relevant relations and controlling attention to competitive, irrelevant information. Halford and colleagues have hypothesized that processing demand increases as the number of relations to be integrated increases (Halford, 1993; Halford, Wilson, & Phillips, 1998). They have proposed a relational complexity rubric for classifying the processing load of analogies by identifying the number of relations and roles that must be processed in parallel to perform structure mapping (Halford, 1993). These authors used a problem introduced by Sweller (1993) to illustrate the distinction between knowledge and relational complexity: "Suppose five days after the day before yesterday is Friday. What day of the week is tomorrow?" This problem is difficult not because we lack the requisite knowledge of the days of the week, but rather because we must process several of the relationships in parallel.

Attempting to learn from analogies with an overwhelming level of relational complexity reduces learners' readiness and ability to make abstract generalizations (Catrambone, 1995, 1996, 1998; Catrambone & Holyoak, 1990). In the example used above between the 1974 naval blockade of Cuba and the hypothetical "blockade" of a computer cyber attack, the reasoner must hold many key relationships simultaneously in working memory in order to make any sensible inferences. In understanding the source analog, it is important to consider the causal structure of a blockade that involved the threat of nuclear missile placements, the physical configuration of the blockade, proximity between the United States and Cuba, and so forth. Similarly, many relationships must be considered in representing the structure of the target analog, such as the causal structure of the threat posed by the virus, the nature of the computer virus, and the potential number of computers effected. Beyond representing these analogs, the reasoner must then hold the relevant relationships active in order to recognize commonalities and differences (e.g., breadth of blockade necessary, budget constraints, threat potential).

The number of potentially relevant relationships easily overwhelms the working memory system, meaning that cognitive readiness to accomplish this mapping requires the use of tools or strategies for reducing this load. Tools that have been shown to reduce working memory load include providing worked examples (Sweller, 1993, 1994; Sweller, van Merriënboer, & Paas, 1998), introducing visuo-spatial representations of the key relations (Kosslyn, 1995), breaking the task into subgoals that can be considered separately (Catrambone & Holyoak, 1989), and using gestures to support reasoning (Goldin-Meadow, 2003).

Inhibitory control is also crucial learning from analogy. The ability to control attention and reduce interference from irrelevant but salient features of analogs allows learners to attend to key structural relationships (Cho, Holyoak, & Cannon, 2007; Richland et al., 2006). In the blockade example this might include inhibiting misleading distinctions between the two situations, such as the difference in appearance between a nuclear missile and computer data or between boats and cyber code. While these objects have many differences in features, many of these properties may not be relevant to drawing inferences from one context to another. In addition, there may be irrelevant similarities between these contexts that might lead a reasoner to a false inference or a misconception.

When the demands on inhibitory control are less pronounced, learners have more resources available to allocate to the task of representing and manipulating key similarities and differences between source and target analogs. Designing reasoning contexts that reduce such demands can thus enhance cognitive readiness. Reducing demands on cognitive resources is particularly feasible and important in formulating training contexts. Because novice reasoners have less complete knowledge of the source domain, they are less able to collapse knowledge into groupings, or "chunks," that can reduce processing demands. In addition, when learners are under stress, their inhibitory control resources are reduced, and they are increasingly likely to reason on the basis of object features (Tohill & Holyoak, 2000). Both training and critical problem-solving environments are often highly stressful, in which case strategies for reducing reasoners' attention to irrelevant object properties may be even more important.

12.4 Techniques for Fostering Effective Learning from Training Analogies: Insights from International Peers

Interventions that reduce demands on learners for working memory and inhibitory control during training enhance their readiness to both retain and transfer concepts taught through analogy. A series of studies, described below, have examined feasible, low resource strategies in instructional contexts involving teaching and learning mathematics. Mathematics learning requires abstract, conceptual knowledge that can be transferred across problem contexts. In particular, this series of studies provides insight into the training strategies of peer countries that tend to outperform American mathematics students on international achievement tests (Gonzales et al., 2008).

One robust and somewhat unexpected finding from international studies using achievement tests and videotapes of instructional practices, the technique that forms the basis for this line of research, has been that teaching techniques seem to be culturally organized. While teachers in the United States differ in some expected ways, different American teachers tend to use very similar practices when compared with teachers from other countries (Hiebert et al., 2003; Stigler & Hiebert, 1999). These similarities imply that teaching is a part of culture, suggesting that changes may be difficult as incorporating new training techniques into current practices takes concerted effort. These findings also mean that studying a relatively small sample of teachers internationally is a valid way to gain insight into cultural norms of training practices.

12.4.1 International Practices in Use of Analogy

Richland, Zur, and Holyoak (2007; see also Richland, Holyoak, & Stigler, 2004) studied a subset of videotaped mathematics lessons collected as part of the Trends in International Mathematics and Science Study (Hiebert et al., 2003). The TIMSS 1999 video study used a random probability sampling method to collect videos of everyday eighth-grade classroom teaching in seven countries internationally that outperform the US students in mathematics and or science. Lessons for study were sampled from all classrooms taught in the country to capture variation across the school year and urban, rural, public, private, and religious levels. Richland et al. (2007) took a random subset of these lessons (ten) from the United States and two higher-achieving Asian regions that both outperform the USA regularly, but also are very different from each other in normative teaching practices: Japan and Hong Kong (China). Japanese lessons tend to center on one or two complex problems that students solve independently and then multiple student solutions are compared to introduce new problem-solving strategies and concepts (Hiebert et al., 2003). By contrast, lessons in Hong Kong tend to follow a model that is more similar to the US lesson structure, with a teacher at the board for much of the lesson who engages the students in working through many smaller problems (Hiebert et al., 2003). All instances of instructional comparisons between source and target analogs were identified in every lesson, and then each of those instances was coded to examine the teacher's strategies for supporting students' readiness to notice and make the relational abstraction from the analogy. Coding was conducted by an international team with native speakers from each country yielding high reliability across all codes.

Codes were developed to reflect teachers' common practices that aligned with the cognitive factors outlined above. These codes sought to capture frequency of instructional decisions that could be expected to encourage learners to draw on prior causal knowledge structures, reduce working memory processing load, and reduce demands on inhibitory control. The codes measured the following practical moves by the teacher: (a) constructing visual and mental imagery, (b) using gestures that moved comparatively between the source and target analogs, (c) spatially aligning written representations of the source and target analogs to highlight structural commonalities, (d) using source analogs likely to have a familiar causal structure to learners, (e) making a visual representation of the source analog visible during comparison with the target, and (f) producing a visual representation of a source analog versus only a verbal one.

The results were clear: Asian teachers used all of these cognitive support strategies reliably more frequently than did American teachers. As shown in Fig. 12.4, some strategies were used frequently, others less often, but the Asian teachers were always more likely to include one or more support strategies with their analogies than were teachers in the United States. These differences in strategy use held in spite of there being no differences in the number of analogies used across countries. Americans teachers used high numbers of analogies, but did not provide the same level of support to aid their students in noticing and learning from the key relational structure of the analogies.

Interestingly, few of the analogies in any country involved a source analog that was likely to be very well known to reasoners. Thus as discussed above, learning was often limited by the students' causal knowledge of the source, as they were acquiring and reasoning about the causal structure of both the source and target analogs simultaneously. This type of learning environment, though seemingly very common across instructional environments in different nations, imposes high cognitive demands. In situations involving learning based on an analogy between a less well-known source and a novel target, providing supports for working memory and inhibitory control resources may be particularly crucial.

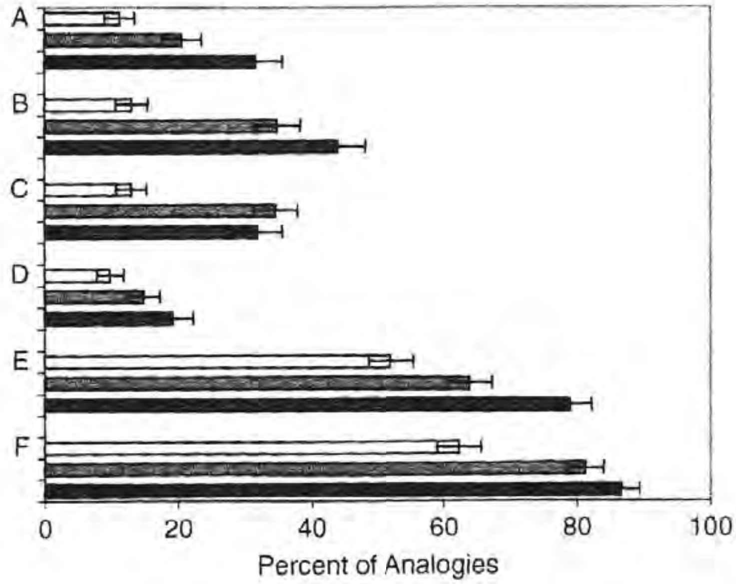

Fig. 12.4 Percent of analogies by region containing cognitive supports: (**a**) visual and mental imagery, (**b**) comparative gesture, (**c**) visual alignment, (**d**) use of a familiar source, (**e**) source visible concurrently with target, (**f**) source presented visually. *White* denotes U.S. teachers, *gray* denotes Chinese teachers, *black* denotes Japanese teachers (from Richland, Zur, & Holyoak, 2007; reprinted by permission)

12.4.2 Experimental Tests of Strategies for Teaching with Analogies

A series of experiments subsequently examined whether using the teaching supports identified in the cross-cultural data described above actually promote learning and transfer when both the source and the target analog are relatively novel. Richland and McDonough (2010) designed instructional videotapes that taught the same two analogs either with or without the support strategies identified in the international video study. Two experiments used materials from the GRE test that were known to be challenging for undergraduates not majoring in a mathematics discipline.

In the first experiment (Richland & McDonough, 2010, Experiment 1), participants were randomly assigned to one of the two conditions: analogy with high cognitive support strategies or analogy with low cognitive support strategies. The content area was permutation and combination problem solving. In both conditions a videotaped teacher first taught and demonstrated a solution to a permutation problem: "Suppose there are five people running in a race. The winner of the race will get a gold medal, the person who comes in second will get a silver medal, and the person who comes in third will get a bronze medal. How many different orders of gold-silver-bronze winners can there be?"

The teacher next taught and demonstrated a solution to a combination problem: "A professor is choosing students to attend a special seminar. She has eleven students to choose from, but she only has four extra tickets available. How many different ways are there to make up the four students chosen to go to the seminar?"

As is evident from these examples, permutation and combination problems share mathematical structure, but there is one key difference. All assigned roles in combination problems are equivalent (e.g., in a combination problem, it does not matter which ticket a student receives). In contrast, order of assignment to roles is critical in permutation problems (e.g., winning gold is different from winning bronze).

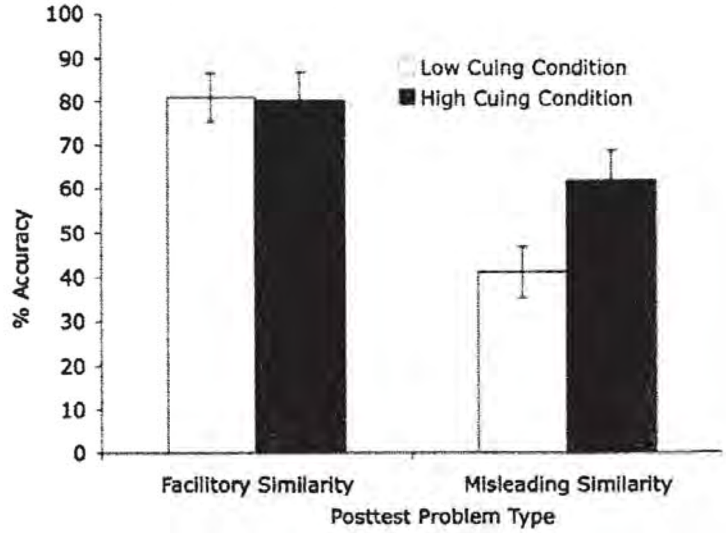

Fig. 12.5 The effects of high versus low cuing of an instructional analogy on posttest problems with varying similarity to instructed problems (from Richland & McDonough, 2010, Experiment 1; reprinted by permission)

In order to solve a combination problem, one divides the total number of permutations by the number of possible role arrangements (in this example problem, 4!, i.e., four factorial, which equals 24 role arrangements).

The posttest was designed to measure both retention of the initial instruction and transfer. The former was assessed using problems with high similarity to the instructed problems, such that permutation problems were set in a race context and combination problems were framed as tickets to a lecture. Transfer was assessed using "misleading similarity" problems in which the surface context and the mathematical structures were cross-mapped; a permutation problem was instantiated as tickets to a lecture, and a combination problem framed as a race. Performance on these problems reflected learners' ability to represent the source and target problem based on mathematical structure as opposed to surface features.

As evident in Fig. 12.5, the data revealed a significant interaction between instructional condition and problem type. There were no differences on the facilitatory similarity problems, revealing that participants in both instructional conditions benefited from the instructional analogy, showing approximately 80 % accuracy on these near-transfer problems (baseline performance with the same population was 7 %). The results were different for the misleading similarity problems (baseline level 10 %). Participants who had received instruction with high support cues were more likely to solve the misleading similarity problems by transferring on the basis of structure versus surface features (62 % accuracy). These participants significantly outperformed participants who had been trained on the low support video, whose accuracy was 41 %. This pattern indicates that either type of instructional analogy was beneficial but that adding supports to aid learners' cognitive readiness led to more adaptable, schematic knowledge representations.

This interaction was replicated in two additional experiments. Richland and McDonough (2010, Experiment 2) obtained a very similar result with undergraduates learning to solve proportion word problems through an analogy between a correct solution and a common but invalid use of the linearity assumption. A third experiment replicated the result in a classroom context with school-age children learning division of rational numbers by analogy to division of natural numbers (Richland & Hansen, in press).

Overall, these data reveal a reliable finding that high quality analogies can be effective tools for enhancing cognitive readiness for learning but that including supplemental strategies to provide cognitive support maximizes the impact of analogy training on transfer. When given such cues, learners seem to have developed more conceptual, schematized representations of the instructed concepts and more adaptive proficiency in representing new problems. These tools require only a small investment on the part of the instructor, yet have the potential for broad learning gains. Importantly, though seemingly quite simple, these strategies for reducing processing load do not appear to be traditional parts of typical training strategies used by teachers in the United States, certainly not in teaching mathematics.

12.5 Conclusions

In general, teaching relational structure constitutes a powerful tool for fostering cognitive readiness for transfer. Instruction based on analogy is not straightforward, however, since limits in relevant knowledge and processing capacity increase the likelihood that learners fail to notice or benefit from analogies in teaching. The aim of the teacher should be to assist the learner in developing veridical causal models of the domain or deep understanding of content structures. Major strategies for using analogies effectively in teaching include guided comparison of examples, highlighting of relations by principles and visual diagrams, ordering examples to encourage progressive alignment, and focusing attention on subgoals.

Other supportive cues can improve acquisition of the underlying relational structure. Useful interventions include reducing processing load, facilitating attention to relational structure of target problems, drawing learners' attention to relations versus object features, reducing competitive interference, and encouraging learners to draw on prior knowledge. The benefits of relational instruction are most apparent when the learner is later faced with novel problems that require extension and adaptation of the earlier examples used in training.

We end with a list of recommendations for practices that can be customized to different learning contexts and training needs. In many training contexts analogies are a widely used, but under-considered, resource for enhancing abstraction and transfer. Comparing a new problem or concept to prior knowledge is a cognitive ability deeply embedded into our thinking and perhaps is what makes human an especially adaptive species (Penn, Holyoak, & Povinelli, 2008). However, explicit training by analogy is not as naturally reliable, and instructional analogies can be greatly improved by using several key support strategies. These tools are maximally important when learners have incomplete knowledge of the domain, are under stress, or are otherwise operating with limited cognitive resources.

12.6 Practical Recommendations for Improving Cognitive Readiness Through Analogy

Training strategies for enhancing cognitive readiness through analogy are collapsed here into a useful list for the reader's reference. Citations and fuller descriptions are available in the text above:

1. Use a source analog with a causal structure that is well known to learners
2. Have learners compare two or more source analogs before transferring to a target
3. Guide learners through comparisons between analogs, either with explicit instructions or mapping tasks
4. Order source analogs from "easy" mappings to more challenging ones, fostering progressive alignment between the analogs
5. Reduce processing load on the working memory system. This can be accomplished in several ways, including:
   (a) Break target problem into subgoals that can be accomplished separately
   (b) Use visual or mental imagery
   (c) Create visual representations of source and target analogs, rather than describing them only verbally
   (d) Make visual representations of source and target analogs visible simultaneously
6. Reduce demands on inhibitory control. This can be accomplished in several ways, including:
   (a) Design source and target visual representations in ways that highlight the key relational correspondences and downplay irrelevant similarities and differences
   (b) Use hand gestures that move between the representations of source and target correspondences to draw attention to relational commonalities

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